# Octonionic Quantum Interplays of Dark Matter and Ordinary Matter

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Based on the electromagnetic interaction and gravitational interaction, the quantum interplays of the ordinary matter and the dark matter in the octonion space are discussed. The paper presents the quantization of the particles of ordinary matter and dark matter, including the quantization of the electromagnetic field and gravitational field etc. In the electromagnetic and gravitational interactions, it deduces some predictions of the field source particle, which are consistent with the Dirac equation and Schrodinger equation in the quantum mechanics. The researches show that there exist some quantum characteristics of the intermediate particle in the electromagnetic and gravitational interactions, including the Dirac-like equation etc.

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## I. INTRODUCTION

At the present time, most scientists believe that the universe abounds with multiform dark matters [1, 2]. A new viewpoint on the problem of dark matter can be given by the concept of octonion space. According to previous research results, the electromagnetic and gravitational interactions can be described by the quaternion space. Based on the conception of space verticality etc., two types of quaternion spaces can be united into an octonion space. In the octonionic space, the electromagnetic and gravitational interactions can be equally described. So the characteristics of the field source particle (electron and proton etc.) and intermediate particle (photon etc.) in the electromagnetic field, gravitational field, and dark matter field can be described by the octonion space uniformly [3].

Quantum mechanics does not deal with the problem of the quantization of the dark matter. For the quantization of the gravitational field and other fields, S. L. Adler [4] etc. developed some new forms of quantum mechanics by means of the quaternion, but they did not consider the case of the dark matter in their researches. And then the puzzle of the quantization of the dark matter remains unclear and has not satisfied results.

The paper extends the quantization for the ordinary matter and dark matter, and draws some conclusions which are consistent with the Dirac equation, Schrodinger equation, and Dirac-like equation etc. A few predictions which are associated with the quantum feature of dark matter can be deduced, and some new and unknown particles can be used for the candidate of dark matter.

## II. ELECTROMAGNETIC-GRAVITATIONAL FIELD

In the electromagnetic-gravitational theory, the physics characteristics of electromagnetic and gravitational fields can be described by the octonion space which is united from a couple of quaternion spaces. And the equations set of the electromagnetic-gravitational field can be attained.

### A. Octonion space

The base $\mathbb{E}_g$ of the quaternion space of the gravitational interaction (G space, for short) is $\mathbb{E}_g = (1,\ \boldsymbol{i}_1,\ \boldsymbol{i}_2,\ \boldsymbol{i}_3)$ . The base $\mathbb{E}_e$ of the quaternion space of the electromagnetic interaction (E space, for short) is independent of the base $\mathbb{E}_g$ . Selecting $\mathbb{E}_e = (1,\ \boldsymbol{i}_1,\ \boldsymbol{i}_2,\ \boldsymbol{i}_3) \circ \boldsymbol{I}_0 = (\boldsymbol{I}_0,\ \boldsymbol{I}_1,\ \boldsymbol{I}_2,\ \boldsymbol{I}_3)$. So the base $\mathbb{E}_g$ and $\mathbb{E}_e$ can constitute the base $\mathbb{E}$ of the octonion space.

$$\mathbb{E} = \mathbb{E}_g + \mathbb{E}_e = (1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3) \tag{1}$$

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TABLE I: The octonion multiplication table.

| | $1$ | $\boldsymbol{i}_1$ | $\boldsymbol{i}_2$ | $\boldsymbol{i}_3$ | $\boldsymbol{I}_0$ | $\boldsymbol{I}_1$ | $\boldsymbol{I}_2$ | $\boldsymbol{I}_3$ |
|---|---|---|---|---|---|---|---|---|
| $1$ | $1$ | $\boldsymbol{i}_1$ | $\boldsymbol{i}_2$ | $\boldsymbol{i}_3$ | $\boldsymbol{I}_0$ | $\boldsymbol{I}_1$ | $\boldsymbol{I}_2$ | $\boldsymbol{I}_3$ |
| $\boldsymbol{i}_1$ | $\boldsymbol{i}_1$ | $-1$ | $\boldsymbol{i}_3$ | $-\boldsymbol{i}_2$ | $\boldsymbol{I}_1$ | $-\boldsymbol{I}_0$ | $-\boldsymbol{I}_3$ | $\boldsymbol{I}_2$ |
| $\boldsymbol{i}_2$ | $\boldsymbol{i}_2$ | $-\boldsymbol{i}_3$ | $-1$ | $\boldsymbol{i}_1$ | $\boldsymbol{I}_2$ | $\boldsymbol{I}_3$ | $-\boldsymbol{I}_0$ | $-\boldsymbol{I}_1$ |
| $\boldsymbol{i}_3$ | $\boldsymbol{i}_3$ | $\boldsymbol{i}_2$ | $-\boldsymbol{i}_1$ | $-1$ | $\boldsymbol{I}_3$ | $-\boldsymbol{I}_2$ | $\boldsymbol{I}_1$ | $-\boldsymbol{I}_0$ |
| $\boldsymbol{I}_0$ | $\boldsymbol{I}_0$ | $-\boldsymbol{I}_1$ | $-\boldsymbol{I}_2$ | $-\boldsymbol{I}_3$ | $-1$ | $\boldsymbol{i}_1$ | $\boldsymbol{i}_2$ | $\boldsymbol{i}_3$ |
| $\boldsymbol{I}_1$ | $\boldsymbol{I}_1$ | $\boldsymbol{I}_0$ | $-\boldsymbol{I}_3$ | $\boldsymbol{I}_2$ | $-\boldsymbol{i}_1$ | $-1$ | $-\boldsymbol{i}_3$ | $\boldsymbol{i}_2$ |
| $\boldsymbol{I}_2$ | $\boldsymbol{I}_2$ | $\boldsymbol{I}_3$ | $\boldsymbol{I}_0$ | $-\boldsymbol{I}_1$ | $-\boldsymbol{i}_2$ | $\boldsymbol{i}_3$ | $-1$ | $-\boldsymbol{i}_1$ |
| $\boldsymbol{I}_3$ | $\boldsymbol{I}_3$ | $-\boldsymbol{I}_2$ | $\boldsymbol{I}_1$ | $\boldsymbol{I}_0$ | $-\boldsymbol{i}_3$ | $-\boldsymbol{i}_2$ | $\boldsymbol{i}_1$ | $-1$ |

The radius vector $\mathbb{R}(r_0, r_1, r_2, r_3, R_0, R_1, R_2, R_3)$ in the octonion space is

$$\mathbb{R} = (r_0 + \boldsymbol{i}_1 r_1 + \boldsymbol{i}_2 r_2 + \boldsymbol{i}_3 r_3) + (\boldsymbol{I}_0 R_0 + \boldsymbol{I}_1 R_1 + \boldsymbol{I}_2 R_2 + \boldsymbol{I}_3 R_3) \tag{2}$$

where, $r_0 = v_0 t$, $R_0 = V_0 T$. $v_0 = V_0 = c$ is the speed of light beam, $t$ and $T$ denote the time.

The octonion algebra is the alternative algebra, so the octonions $\mathbb{Q}_1$ and $\mathbb{Q}_2$ satisfy

$$\mathbb{Q}_1 \circ (\mathbb{Q}_1 \circ \mathbb{Q}_2) = (\mathbb{Q}_1 \circ \mathbb{Q}_1) \circ \mathbb{Q}_2 \ , \quad \mathbb{Q}_1 \circ (\mathbb{Q}_2 \circ \mathbb{Q}_2) = (\mathbb{Q}_1 \circ \mathbb{Q}_2) \circ \mathbb{Q}_2 \ .$$

The octonion differential operator $\lozenge$ and its conjugate operator $\lozenge^*$ are defined as

$$\lozenge = \lozenge_g + \lozenge_e \ . \quad \lozenge_g = \partial_{g0} + \boldsymbol{i}_1 \partial_{g1} + \boldsymbol{i}_2 \partial_{g2} + \boldsymbol{i}_3 \partial_{g3} \ ; \quad \lozenge_e = \boldsymbol{I}_0 \partial_{e0} + \boldsymbol{I}_1 \partial_{e1} + \boldsymbol{I}_2 \partial_{e2} + \boldsymbol{I}_3 \partial_{e3} \ . \tag{3}$$

where, $\partial_{gi} = \partial / \partial r_i$ ; $\partial_{ei} = \partial / \partial R_i$ ; $i = 0, 1, 2, 3$ .

The field potential $\mathbb{A}(a_0, a_1, a_2, a_3, A_0, A_1, A_2, A_3)$ in the electromagnetic-gravitational field is defined as

$$\mathbb{A} = \lozenge^* \circ \mathbb{X} = (a_0 + \boldsymbol{i}_1 a_1 + \boldsymbol{i}_2 a_2 + \boldsymbol{i}_3 a_3) + k_a (\boldsymbol{I}_0 A_0 + \boldsymbol{I}_1 A_1 + \boldsymbol{I}_2 A_2 + \boldsymbol{I}_3 A_3) \tag{4}$$

where, $\mathbb{X} = \mathbb{X}_g + k_{rx} \mathbb{X}_e$ ; $\mathbb{X}_g$ and $\mathbb{X}_e$ are physical quantities in G space and in E space respectively; $\mathbb{A} = \mathbb{A}_g + k_a \mathbb{A}_e$; $\mathbb{A}_g = (a_0, a_1, a_2, a_3)$ and $\mathbb{A}_e = (A_0, A_1, A_2, A_3)$ are the field potential in G space and in E space respectively; $k_a$ and $k_{rx}$ are the coefficients.

### B. Dark matter

In the electromagnetic-gravitational field, there exist four types of subfields and their field sources. In Table 2, the electromagnetic-gravitational (E-G) subfield and gravitational-gravitational (G-G) subfield are 'electromagnetic field' and 'gravitational field' respectively. And their general charges are electronic charge (G charge) and the mass (G mass) respectively. The electromagnetic-electromagnetic (E-E) and gravitational-electromagnetic (G-E) subfields are both long range fields and candidates of the 'dark matter field'. Their general charges (E charge and E mass) are candidates of 'dark matter'. The physical features of the dark matter meet the requirements of Eqs.(4)$-$(10).

In the general charge $(Q_e^e, Q_e^g, Q_g^e, Q_g^g)$ and intermediate particle $(\gamma_e^e, \gamma_e^g, \gamma_g^e, \gamma_g^g)$, two types of general charges (the electronic charge $Q_e^g$, the mass $Q_g^g$) and one type of intermediate particle (the photon $\gamma_e^g$) have been found. Therefore the two types of general charges and three types of intermediate particles are left to be found in Table 2.

The particles of the ordinary matter (the electron and proton etc.) possess the G charge together with G mass. The particles of the dark matter may possess the E charge with E mass, or G mass with E charge, etc. It can be predicted that the field strength of the electromagnetic-electromagnetic and the gravitational-electromagnetic subfields must be weaker than that of the gravitational-gravitational subfield, otherwise they should be detected for a long time. So the field strength of the electromagnetic-electromagnetic subfield and gravitational-electromagnetic subfield may be equal, and both of them are slightly weaker than that of the gravitational-gravitational subfield.

### C. Equations sets

In the electromagnetic-gravitational field, the force $\mathbb{Z}$ and energy $\mathbb{W}$ can be defined uniformly, and the octonion differential operator $\lozenge$ needs to be generalized to the $(\lozenge + \mathbb{B}/\alpha)$. So the physical characteristics of the electromagnetic-gravitational field can be researched from many aspects.

TABLE II: The subfield types of electromagnetic-gravitational field.

| Operator | Gravitational Interaction | Electromagnetic Interaction |
|---|---|---|
| operator $\lozenge_g$ | gravitational-gravitational subfield | electromagnetic-gravitational subfield |
| of G space | (gravitational field, or G-G subfield) | (electromagnetic field, or E-G subfield) |
| | G mass, $Q_g^g$ | G charge, $Q_e^g$ |
| | intermediate particle, $\gamma_g^g$ | intermediate particle, $\gamma_e^g$ |
| | long range field, weak strength | long range field, strong strength |
| operator $\lozenge_e$ | gravitational-electromagnetic subfield | electromagnetic-electromagnetic subfield |
| of E space | (dark matter field, or G-E subfield) | (dark matter field, or E-E subfield) |
| | E mass, $Q_g^e$ | E charge, $Q_e^e$ |
| | intermediate particle, $\gamma_g^e$ | intermediate particle, $\gamma_e^e$ |
| | long range field, weaker strength | long range field, weaker strength |

The field strength $\mathbb{B}(b_0, b_1, b_2, b_3, B_0, B_1, B_2, B_3)$ of electromagnetic-gravitational field can be defined as

$$\mathbb{B} = \lozenge \circ \mathbb{A} \tag{5}$$

where, $\mathbb{B} = \mathbb{B}_g + k_b \mathbb{B}_e$ ; $\mathbb{B}_g = (b_1, b_2, b_3)$ and $\mathbb{B}_e = (B_1, B_2, B_3)$ are respectively the field strength in G space and in E space. Selecting the gauge equation, $b_0 = 0$ and $B_0 = 0$, can simplify definition of field strength.

The field source and the force of the electromagnetic-gravitational field can be defined as follow respectively. The mark (*) denotes octonion conjugate. ($\alpha = c$ is a coefficient )

$$\mu \mathbb{S} = (\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{B} \tag{6}$$

$$\mathbb{Z} = \alpha (\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{P} \tag{7}$$

The definition of force shows that, the force $\mathbb{Z}$ , field source $\mathbb{S}$ , and linear momentum $\mathbb{P} = \mu \mathbb{S} / \mu_g^g$ need to be revised and generalized to the form in above equations. As a part of field source $\mathbb{S}$, the term $(\mathbb{B}^* \circ \mathbb{B} / \mu_g^g)$ includes the field energy density. When the force $\mathbb{Z} = 0$, we obtain the force-balance equation of the electromagnetic-gravitational field.

The angular momentum in the electromagnetic-gravitational field can be defined as,

$$\mathbb{M} = (\mathbb{R} + k_{rx} \mathbb{X}) \circ \mathbb{P} \tag{8}$$

where, $k_{rx}$ is a coefficient .

And the energy and power in the electromagnetic-gravitational field can be defined as

$$\mathbb{W} = \alpha (\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{M} \tag{9}$$

$$\mathbb{N} = \alpha (\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{W} \tag{10}$$

The extended definition of the energy shows that, the angular momentum $\mathbb{M}$, the energy $\mathbb{W}$ and the power $\mathbb{N}$ need to be revised and generalized to the form in the above equations. The physical quantity $\mathbb{X}$ has effect on the field potential $\mathbb{A} = \lozenge^* \circ \mathbb{X}$ , the angular momentum $\mathbb{X} \circ \mathbb{P}$ , the energy $\mathbb{B}^* \circ (\mathbb{X} \circ \mathbb{P})$ and the power $(\mathbb{B} \circ \mathbb{B}^*) \circ (\mathbb{X} \circ \mathbb{P})$ . The introduction of physical quantity $\mathbb{X}$ makes the definition of the angular momentum $\mathbb{M}$ and the energy $\mathbb{W}$ more integrated, and the theory more self-consistent.

In the above equations, the conservation of angular momentum in the electromagnetic-gravitational field can be gained when $\mathbb{W} = 0$, and the energy conservation equation in the electromagnetic-gravitational field can be attained when $\mathbb{N} = 0$.

TABLE III: The comparison between the ordinary matter fields with dark matter fields.

| | Ordinary Matter Fields | | Dark Matter Fields | |
|---|---|---|---|---|
| (field) | (gravitational field) | (electromagnetic field) | (dark matter field) | (dark matter field) |
| subfield | G-G subfield | E-G subfield | G-E subfield | E-E subfield |
| field potential | $\lozenge_g^* \circ \mathbb{X}_g$ | $\lozenge_g^* \circ \mathbb{X}_e$ | $\lozenge_e^* \circ \mathbb{X}_g$ | $\lozenge_e^* \circ \mathbb{X}_e$ |
| field strength | $\lozenge_g \circ \mathbb{A}_g$ | $\lozenge_g \circ \mathbb{A}_e$ | $\lozenge_e \circ \mathbb{A}_g$ | $\lozenge_e \circ \mathbb{A}_e$ |
| field source | $\lozenge_g^* \circ \mathbb{B}_g$ | $\lozenge_g^* \circ \mathbb{B}_e$ | $\lozenge_e^* \circ \mathbb{B}_g$ | $\lozenge_e^* \circ \mathbb{B}_e$ |

## III. EQUATIONS OF QUANTUM MECHANICS

### A. Dirac Equation

In the octonion space, the wave functions of the quantum mechanics are the octonion equations set. And the Dirac equation of the quantum mechanics are actually the wave equations set which are associated with the particle's wave function $-\boldsymbol{I} \circ \mathbb{M}/\hbar$ .

The $\mathbb{U}$ equation of the quantum mechanics can be defined as

$$\mathbb{U} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (-\boldsymbol{I} \circ \mathbb{M}/\hbar) \tag{11}$$

where, $-\boldsymbol{I} \circ \mathbb{M}/\hbar$ is the wave function for particles of the field source; $\boldsymbol{I}$ is the octonion unit, $\boldsymbol{I}^* \circ \boldsymbol{I} = 1$; the coefficient $h$ is the Planck constant, and $\hbar = h/2\pi$ .

The $\mathbb{L}$ equation of the quantum mechanics can be defined as

$$\mathbb{L} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{U}/\hbar) \tag{12}$$

From the equation $\mathbb{U} = 0$, the Dirac and Schrodinger equations in the electromagnetic-gravitational field can be deduced to describe the field source particle (electron and proton etc.) with spin $1/2$ . Those equations can conclude the results which are consistent with the wave equations for particles of the field source in certain cases [5, 6, 7].

### B. Dirac-like Equation

Through the comparison, we find that Dirac equation of Eq.(11) and Eq.(12) can be attained respectively from the energy equation Eq.(9) and power equation Eq.(10) after substituting the operator $\alpha(\mathbb{B}/\alpha + \lozenge)$ for $\{\mathbb{W}/(\alpha\hbar) + \lozenge\}$ . By analogy with the above equations, the Dirac-like equations of Eq.(13) and (14) can be obtained from the field source equation Eq.(6) and force equation Eq.(7) respectively.

The $\mathbb{T}$ equation of the quantum mechanics can be defined as

$$\mathbb{T} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) \tag{13}$$

where, $\mathbb{B}/\hbar$ is the wave function for the intermediate particle.

The $\mathbb{O}$ equation of the quantum mechanics can be defined as

$$\mathbb{O} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{T}/\hbar) \tag{14}$$

From the equation $\mathbb{T} = 0$, the Dirac-like equation in electromagnetic-gravitational field can be deduced to describe the intermediate particle (photon etc.) with spin 1 . Those equations can conclude results which are consistent with the wave function for the intermediate particle in certain cases [8, 9].

TABLE IV: The summary of main definitions and equations.

| $\mathbb{X}$ physical quantity | $\mathbb{X}$ |
|---|---|
| Field potential | $\mathbb{A} = \lozenge^* \circ \mathbb{X}$ |
| Field strength | $\mathbb{B} = \lozenge \circ \mathbb{A}$ |
| Field source | $\mu\mathbb{S} = (\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{B}$ |
| Force | $\mathbb{Z} = \alpha(\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{P}$ |
| Angular momentum | $\mathbb{M} = (\mathbb{R} + k_{rx}\mathbb{X}) \circ \mathbb{P}$ |
| Energy | $\mathbb{W} = \alpha(\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{M}$ |
| Power | $\mathbb{N} = \alpha(\mathbb{B}/\alpha + \lozenge)^* \circ \mathbb{W}$ |
| Energy quantum | $\mathbb{U} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (-\boldsymbol{I} \circ \mathbb{M}/\hbar)$ |
| Power quantum | $\mathbb{L} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{U}/\hbar)$ |
| Field source quantum | $\mathbb{T} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar)$ |
| Force quantum | $\mathbb{O} = (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{T}/\hbar)$ |

### C. Wave function

The wave function $-\boldsymbol{I} \circ \mathbb{M}/\hbar$ is that of the field source particle (electron and proton etc.), and its square module represents the probability density of field source particle. The wave function $\mathbb{B}/\hbar$ is that of the wave function of intermediate particle (photon etc.), and its square module represents the probability density of intermediate particle.

The octonion physical quantity $\mathbb{Q}(q_0, q_1, q_2, q_3, Q_0, Q_1, Q_2, Q_3)$ can be written as exponential format

$$\begin{aligned}\mathbb{Q} &= (q_0 + \boldsymbol{i}_1 q_1 + \boldsymbol{i}_2 q_2 + \boldsymbol{i}_3 q_3) + (\boldsymbol{I}_0 Q_0 + \boldsymbol{I}_1 Q_1 + \boldsymbol{I}_2 Q_2 + \boldsymbol{I}_3 Q_3) \\ &= (q_0 + \boldsymbol{i}_1 q_1 + \boldsymbol{i}_2 q_2 + \boldsymbol{i}_3 q_3) + (\boldsymbol{i}_0 Q_0 + \boldsymbol{i}_1 Q_1 + \boldsymbol{i}_2 Q_2 + \boldsymbol{i}_3 Q_3) \circ \boldsymbol{I}_0 \\ &= q\ exp(\boldsymbol{q}_1 \omega_1) + Q\ exp(\boldsymbol{q}_2 \omega_2) \circ \boldsymbol{I}_0 \end{aligned} \tag{15}$$

where, $q$ and $Q$ are the modules; $\omega_1$ and $\omega_2$ are the angles; $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are the unit vectors in the quaternion G space.

In the quaternion space, the quaternion product of the coefficient $1/\hbar$ , momentum $\mathbb{P}(p_0, p_1, p_2, p_3)$ and radius vector $\mathbb{R}(r_0, r_1, r_2, r_3)$ is

$$\mathbb{M}/\hbar = \mathbb{R} \circ \mathbb{P}/\hbar = A(cos\theta + \boldsymbol{I} sin\theta) = A\ exp(\boldsymbol{I}\theta) \tag{16}$$

where, $\boldsymbol{I}$ is the quaternion unit, $\boldsymbol{I}^* \circ \boldsymbol{I} = 1$ , $A$ is the amplitude; $\mathbb{X} \approx 0$ .

The scalar quantity $\nu_0$ of the quaternion $\mathbb{M}/\hbar$ is

$$\nu_0 = A\ cos\theta = (p_0 r_0 - p_1 r_1 - p_2 r_2 - p_3 r_3)/\hbar \tag{17}$$

where, $p_0 = mc$ , $r_0 = ct$.

When $\nu_0/A \ll 1$, the angle can be written as the expansion of Taylor progression

$$\theta = arccos(\nu_0/A) \approx \pi/2 - \nu_0/A \tag{18}$$

therefore we have the wave function

$$\mathbb{M}/\hbar = \mathbb{R} \circ \mathbb{P}/\hbar = A\ \boldsymbol{I} \circ exp(-\boldsymbol{I}\,\nu_0/A) \tag{19}$$

The quaternion wave function can also be defined as

$$\Psi = -\boldsymbol{I} \circ (\mathbb{M}/\hbar) = A\ exp\{-\boldsymbol{I}\,(p_0 r_0 - p_1 r_1 - p_2 r_2 - p_3 r_3)/\hbar\} \tag{20}$$

The above means that the matter can be represented as either the particle or the wave in the quaternion space or octonion space. In certain cases, the quaternion wave function can be written as the four-component or exponential format. If its direction could be neglected, the quaternion unit $\boldsymbol{I}$ would be substituted with the imaginary unit $i$. And a couple of quaternion wave functions can be written as the eight-component or exponential format in the octonion space in the same way as Eq.(15).

## IV. QUANTIZATION OF ORDINARY MATTER

In the electromagnetic-gravitational field, there is only one sort of the ordinary matter with a pair of general charges. Their Dirac and Schrodinger equations can be attained when $\mathbb{U} = 0$ from Eq.(11). And the Dirac-like equation can be obtained when $\mathbb{T} = \mathbb{W} = 0$ from Eq.(13). With the characteristics of those equations, we can investigate the quantum properties of the field source particle and intermediate particle.

### A. Dirac and Schrodinger equations

In the octonion space, the electromagnetic-gravitational subfield (electromagnetic field) and gravitational-gravitational subfield (gravitational field) are generated by the physical object $M$ which owns rotation and charge. The G current and G momentum of the field source particle $N(m, q)$ are $(S_0^g, S_1^g, S_2^g, S_3^g)$ and $(s_0^g, s_1^g, s_2^g, s_3^g)$ respectively. The $N(m, q)$ is the mixture of the general charges $Q_g^g$ (the mass, $m$) and $Q_e^g$ (the electric charge, $q$). When $\mathbb{U} = 0$ , the wave equation of the particle $N(m, q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\Diamond)^* \circ (-\boldsymbol{I} \circ \mathbb{M}/\hbar) = 0 \tag{21}$$

Because of $|S_0^g| \gg |S_i^g|$ and $|s_0^g| \gg |s_i^g|$ , then $(i=1,2,3)$

$$\begin{aligned}\mathbb{W} &= (\mathbb{B}+\alpha\lozenge)^* \circ \{(\mathbb{R}+k_{rx}\mathbb{X})\circ\mathbb{P}\}\\ &\approx \alpha\lozenge^*\circ\left\{(\mathbb{R}+k_{rx}\mathbb{X})\circ(\mu_g^g s_0^g + k_b\mu_e^g \boldsymbol{I}_0 S_0^g)\right\}/\mu_g^g + \mathbb{B}^*\circ\mathbb{M}\\ &\approx \alpha(m\mathbb{V}+q\mathbb{A}') + \mathbb{B}^*\circ\mathbb{M} + \alpha k_b\mu_e^g S_0^g\left\{\lozenge^*\circ(\mathbb{R}\circ\boldsymbol{I}_0)\right\}/\mu_g^g + k_{rx}s_0^g\mathbb{A}\end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}k_b\mu_e^g/\mu_g^g)\lozenge^*\circ(\mathbb{X}\circ\boldsymbol{I}_0) = a'_0 + \boldsymbol{i}_1 a'_1 + \boldsymbol{i}_2 a'_2 + \boldsymbol{i}_3 a'_3 + \boldsymbol{I}_0 A'_0 + \boldsymbol{I}_1 A'_1 + \boldsymbol{I}_2 A'_2 + \boldsymbol{I}_3 A'_3$; $S_0^g = qc, s_0^g = mc$; $\mathbb{V} = v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3 + \boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3$ ; $m$ is the $Q_g^g$, $q$ is the $Q_e^g$ .

When the $\mathbb{B}$ is small, and the sum of last three terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned}\mathbb{W}/\alpha = &\ m\mathbb{V}+q\mathbb{A}'\\ = &\ (qa'_0+mv_0) + \boldsymbol{i}_1(qa'_1+mv_1) + \boldsymbol{i}_2(qa'_2+mv_2) + \boldsymbol{i}_3(qa'_3+mv_3)\\ &+\boldsymbol{I}_0(qA'_0+mV_0) + \boldsymbol{I}_1(qA'_1+mV_1) + \boldsymbol{I}_2(qA'_2+mV_2) + \boldsymbol{I}_3(qA'_3+mV_3)\\ = &\ p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3\end{aligned} \tag{22}$$

where, $p_j = qa'_j + mv_j$ ; $P_j = qA'_j + mV_j$ ; $j = 0,1,2,3$ .

Then

$$\begin{aligned}0 = &\ (\mathbb{W}/\alpha+\hbar\lozenge)^*\circ\{(\mathbb{W}/\alpha+\hbar\lozenge)^*\circ\Psi\}\\ \approx &\ [(p_0+\hbar\partial_{g0})^2 - (p_1+\hbar\partial_{g1})^2 - (p_2+\hbar\partial_{g2})^2 - (p_3+\hbar\partial_{g3})^2 - (P_0+\hbar\partial_{e0})^2\\ &-(P_1+\hbar\partial_{e1})^2 - (P_2+\hbar\partial_{e2})^2 - (P_3+\hbar\partial_{e3})^2 + q\hbar\lozenge^*\circ\mathbb{A}'^* + m\hbar\lozenge^*\circ(\boldsymbol{I}_0\circ\mathbb{V})^*]\circ\Psi\end{aligned} \tag{23}$$

where, $\Psi = -\boldsymbol{I}\circ\mathbb{M}/\hbar$ is the wave function.

In the above equation, the conservation of wave function is influenced by the field potential, field strength, field source, velocity, charge and mass etc. When the linear momentum $P_j$ is equal approximately to 0, and the wave function is $\Psi = \psi(r)exp(-\boldsymbol{I}\,Et/\hbar)$, the above equation can be simplified as

$$\begin{aligned}0 = &\ [(p_0 - \boldsymbol{I}\,E/c)^2 - (p_1)^2 - (p_2)^2 - (p_3)^2 + q\hbar\lozenge^*\circ\mathbb{A}'^*]\circ\psi(r)\\ \approx &\ [(qa'_0 - \boldsymbol{I}\,E/c) - (1/2mc)\left\{(p_1)^2+(p_2)^2+(p_3)^2\right\} + (q\hbar/2mc)(\lozenge^*\circ\mathbb{A}'^*)]\circ\psi(r)\end{aligned} \tag{24}$$

where, $E$ is the energy; $(q\hbar/2m)(\lozenge^*\circ\mathbb{A}'^*)$ is the interplay term of the electromagnetic-gravitational subfield with the spin $(q\hbar/2m)$.

Limited within certain conditions, Eq.(11) of the electromagnetic-gravitational field in the octonion space can draw some conclusions which are consistent with Dirac and Schrodinger equations about ordinary matter, including the spin and magnetic moment etc.

### B. Intermediate particle equations

In the octonion space, the electromagnetic-gravitational subfield and gravitational-gravitational subfield are generated by the physical object $M$ which owns rotation and charge. The G current and G momentum of the intermediate particle $N(m,q)$ are $(S_0^g, S_1^g, S_2^g, S_3^g)$ and $(s_0^g, s_1^g, s_2^g, s_3^g)$ respectively. The $N(m,q)$ is the mixture of the intermediate particles $\gamma_e^g$ (the photon) and $\gamma_g^g$ . When $\mathbb{T}=0$ , the wave equation of the particle $N(m,q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha+\hbar\lozenge)^*\circ(\mathbb{B}/\hbar) = 0 \tag{25}$$

When the $\mathbb{B}$ is small, and the sum of last three terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned}\mathbb{W}/\alpha = &\ m\mathbb{V}+q\mathbb{A}'\\ = &\ (qa'_0+mv_0) + \boldsymbol{i}_1(qa'_1+mv_1) + \boldsymbol{i}_2(qa'_2+mv_2) + \boldsymbol{i}_3(qa'_3+mv_3)\\ &+\boldsymbol{I}_0(qA'_0+mV_0) + \boldsymbol{I}_1(qA'_1+mV_1) + \boldsymbol{I}_2(qA'_2+mV_2) + \boldsymbol{I}_3(qA'_3+mV_3)\\ = &\ p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3\end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}k_b\mu_e^g/\mu_g^g)\lozenge^*\circ(\mathbb{X}\circ\boldsymbol{I}_0) = a'_0 + \boldsymbol{i}_1 a'_1 + \boldsymbol{i}_2 a'_2 + \boldsymbol{i}_3 a'_3 + \boldsymbol{I}_0 A'_0 + \boldsymbol{I}_1 A'_1 + \boldsymbol{I}_2 A'_2 + \boldsymbol{I}_3 A'_3$; $S_0^g = qc, s_0^g = mc$; $\mathbb{V} = v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3 + \boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3$ ; $p_j = qa'_j + mv_j$ ; $P_j = qA'_j + mV_j$ ; $j = 0,1,2,3$.

Then

$$\begin{aligned}0 = \quad & (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \{(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \Psi\} \\ \approx \quad & [(p_0 + \hbar\partial_{g0})^2 - (p_1 + \hbar\partial_{g1})^2 - (p_2 + \hbar\partial_{g2})^2 - (p_3 + \hbar\partial_{g3})^2 - (P_0 + \hbar\partial_{e0})^2 \\ & -(P_1 + \hbar\partial_{e1})^2 - (P_2 + \hbar\partial_{e2})^2 - (P_3 + \hbar\partial_{e3})^2 + q\hbar\lozenge^* \circ \mathbb{A}'^* + m\hbar\lozenge^* \circ (\boldsymbol{I}_0 \circ \mathbb{V})^*] \circ \Psi \end{aligned} \tag{26}$$

where, $\Psi = \mathbb{B}/\hbar$ is the wave function; $(q\hbar/m)(\lozenge^* \circ \mathbb{A}'^*)$ is the interplay term of the electromagnetic-gravitational subfield with the spin $(q\hbar/m)$.

The above equation can be used to describe the quantum characteristics of intermediate particles which possess the spin 1, G charge and G mass. Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can deduce the wave equation of ordinary matter and its conclusions.

### C. Dirac-like equation

In the octonion space, the electromagnetic-gravitational subfield and gravitational-gravitational subfield are generated by the physical object $M$ which owns rotation and charge. The G current and G momentum of the intermediate particle $N(m, q)$ are $(S_0^g, S_1^g, S_2^g, S_3^g)$ and $(s_0^g, s_1^g, s_2^g, s_3^g)$ respectively. The $N(m, q)$ is the mixture of the intermediate particles $\gamma_g^g$ and $\gamma_e^g$ . When $\mathbb{T} = 0$ , the wave equation of the particle $N(m, q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) = 0 \tag{27}$$

When the energy $\mathbb{W} = 0$, the Dirac-like equation can be attained from the above equation

$$\hbar\lozenge^* \circ (\mathbb{B}/\hbar) = 0 \tag{28}$$

Dirac equation can conclude that field source particles (electron and proton etc.) possess the spin $(q\hbar/2m)$. In the same way, the above equation can infer that intermediate particles (photon etc.) own the spin $(q\hbar/m)$ and have no G charge nor G mass. And the familiar quantum theory of electromagnetic field can be obtained [10, 11, 12].

Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can draw some conclusions which are consistent with Dirac-like equation about ordinary matter.

## V. QUANTIZATION OF DARK MATTER A

In the electromagnetic-gravitational field, there are five sorts of dark matters with a pair of general charges. Two of them will be discussed here. With the characteristics of Eqs.(11) and (13), we can investigate the quantum interplays among the electromagnetic-gravitational subfield (ordinary matter field) with electromagnetic-electromagnetic subfield (dark matter field), and describe their quantum properties of the field source particle and intermediate particle.

### A. Dirac and Schrodinger equations

In the octonion space, the electromagnetic-gravitational subfield and electromagnetic-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E current and G current of the field source particle $N(d, q)$ are $(S_0^e, S_1^e, S_2^e, S_3^e)$ and $(S_0^g, S_1^g, S_2^g, S_3^g)$ respectively. The $N(d, q)$ is the mixture of the general charges $Q_e^e$ (the dark matter mass, $d$) and $Q_e^g$ (the electric charge, $q$). When $\mathbb{U} = 0$, the wave equation of particle $N(d, q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (-\boldsymbol{I} \circ \mathbb{M}/\hbar) = 0 \tag{29}$$

Because of $|S_0^g| \gg |S_i^g|$ and $|S_0^e| \gg |S_i^e|$ , then $(i = 1, 2, 3)$

$$\begin{aligned}\mathbb{W} \quad & = (\mathbb{B} + \alpha\lozenge)^* \circ \{(\mathbb{R} + k_{rx}\mathbb{X}) \circ \mathbb{P}\} \\ & \approx \alpha\lozenge^* \circ \{(\mathbb{R} + k_{rx}\mathbb{X}) \circ (k_b\mu_e^e S_0^e + k_b\mu_e^g \boldsymbol{I}_0 S_0^g)\}/\mu_g^g + \mathbb{B}^* \circ \mathbb{M} \\ & \approx \alpha(d\mathbb{V}' + q\mathbb{A}') + \mathbb{B}^* \circ \mathbb{M} + \alpha k_b[\mu_e^g S_0^g \{\lozenge^* \circ (\mathbb{R} \circ \boldsymbol{I}_0)\} + \mu_e^e k_{rx} S_0^e \mathbb{A}]/\mu_g^g\end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}k_b\mu_e^g/\mu_g^g)\lozenge^* \circ (\mathbb{X} \circ \boldsymbol{I}_0) = a_0' + \boldsymbol{i}_1 a_1' + \boldsymbol{i}_2 a_2' + \boldsymbol{i}_3 a_3' + \boldsymbol{I}_0 A_0' + \boldsymbol{I}_1 A_1' + \boldsymbol{I}_2 A_2' + \boldsymbol{I}_3 A_3'$; $S_0^g = qc, S_0^e = dc$; $\mathbb{V}' = (k_b\mu_e^e/\mu_g^g)\mathbb{V} = v_0' + \boldsymbol{i}_1 v_1' + \boldsymbol{i}_2 v_2' + \boldsymbol{i}_3 v_3' + \boldsymbol{I}_0 V_0' + \boldsymbol{I}_1 V_1' + \boldsymbol{I}_2 V_2' + \boldsymbol{I}_3 V_3'$ ; $q$ is the $Q_e^g$, $d$ is the $Q_e^e$ .

When the $\mathbb{B}$ is small, and the sum of last two terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned}\mathbb{W}/\alpha = & \ d\mathbb{V}' + q\mathbb{A}' \\ = & \ (qa'_0 + dv'_0) + \boldsymbol{i}_1(qa'_1 + dv'_1) + \boldsymbol{i}_2(qa'_2 + dv'_2) + \boldsymbol{i}_3(qa'_3 + dv'_3) \\ & + \boldsymbol{I}_0(qA'_0 + dV'_0) + \boldsymbol{I}_1(qA'_1 + dV'_1) + \boldsymbol{I}_2(qA'_2 + dV'_2) + \boldsymbol{I}_3(qA'_3 + dV'_3) \\ = & \ p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3\end{aligned}$$

where, $p_j = qa'_j + dv'_j$ ; $P_j = qA'_j + dV'_j$ ; $j = 0, 1, 2, 3$.

Therefore

$$\begin{aligned}0 = & \ (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \{(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \Psi\} \\ \approx & \ [(p_0 + \hbar\partial_{g0})^2 - (p_1 + \hbar\partial_{g1})^2 - (p_2 + \hbar\partial_{g2})^2 - (p_3 + \hbar\partial_{g3})^2 - (P_0 + \hbar\partial_{e0})^2 \\ & - (P_1 + \hbar\partial_{e1})^2 - (P_2 + \hbar\partial_{e2})^2 - (P_3 + \hbar\partial_{e3})^2 + q\hbar\lozenge^* \circ \mathbb{A}'^* + d\hbar\lozenge^* \circ (\boldsymbol{I}_0 \circ \mathbb{V}')^*] \circ \Psi\end{aligned} \tag{30}$$

where, $\Psi = -\boldsymbol{I} \circ \mathbb{M}/\hbar$ is the wave function.

In the above equation, the conservation of wave function is influenced by the field potential, field strength, field source, velocity, charge and mass etc. When the linear momentum $P_j$ is equal approximately to 0, and the wave function is $\Psi = \psi(r)exp(-\boldsymbol{I}\, E_d t/\hbar)$, the above equation can be simplified as

$$\begin{aligned}0 = & \ [(p_0 - \boldsymbol{I}\, E_d/c)^2 - (p_1)^2 - (p_2)^2 - (p_3)^2 + q\hbar\lozenge^* \circ \mathbb{A}'^*] \circ \psi(r) \\ \approx & \ [(qa'_0 - \boldsymbol{I}\, E_d/c) - (1/2dc)\left\{(p_1)^2 + (p_2)^2 + (p_3)^2\right\} + (q\hbar/2dc)(\lozenge^* \circ \mathbb{A}'^*)] \circ \psi(r)\end{aligned} \tag{31}$$

where, $E_d$ is the 'energy'; $(q\hbar/2d)(\lozenge^* \circ \mathbb{A}'^*)$ is the interplay term of the electromagnetic-gravitational subfield with the spin $(q\hbar/2d)$.

Limited within certain conditions, Eq.(11) of the electromagnetic-gravitational field in the octonion space can deduce the Dirac and Schrodinger equations and their conclusions about interplays of dark matter and ordinary matter, including the spin and magnetic moment etc.

### B. Intermediate particle equations

In the octonion space, the electromagnetic-gravitational subfield and electromagnetic-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E current and G current of the intermediate particle $N(d, q)$ are $(S^e_0, S^e_1, S^e_2, S^e_3)$ and $(S^g_0, S^g_1, S^g_2, S^g_3)$ respectively. The $N(d, q)$ is the mixture of the intermediate particles $\gamma^g_e$ (the photon) and $\gamma^e_e$ . When $\mathbb{T} = 0$, the wave equation of particle $N(d, q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) = 0 \tag{32}$$

When the $\mathbb{B}$ is small, and the sum of last two terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned}\mathbb{W}/\alpha = & \ d\mathbb{V}' + q\mathbb{A}' \\ = & \ (qa'_0 + dv'_0) + \boldsymbol{i}_1(qa'_1 + dv'_1) + \boldsymbol{i}_2(qa'_2 + dv'_2) + \boldsymbol{i}_3(qa'_3 + dv'_3) \\ & + \boldsymbol{I}_0(qA'_0 + dV'_0) + \boldsymbol{I}_1(qA'_1 + dV'_1) + \boldsymbol{I}_2(qA'_2 + dV'_2) + \boldsymbol{I}_3(qA'_3 + dV'_3) \\ = & \ p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3\end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}k_b\mu^g_e/\mu^g_g)\lozenge^* \circ (\mathbb{X} \circ \boldsymbol{I}_0) = a'_0 + \boldsymbol{i}_1 a'_1 + \boldsymbol{i}_2 a'_2 + \boldsymbol{i}_3 a'_3 + \boldsymbol{I}_0 A'_0 + \boldsymbol{I}_1 A'_1 + \boldsymbol{I}_2 A'_2 + \boldsymbol{I}_3 A'_3$; $S^g_0 = qc, S^e_0 = dc$; $\mathbb{V}' = (k_b\mu^e_e/\mu^g_g)\mathbb{V} = v'_0 + \boldsymbol{i}_1 v'_1 + \boldsymbol{i}_2 v'_2 + \boldsymbol{i}_3 v'_3 + \boldsymbol{I}_0 V'_0 + \boldsymbol{I}_1 V'_1 + \boldsymbol{I}_2 V'_2 + \boldsymbol{I}_3 V'_3$ ; $p_j = qa'_j + dv'_j$ ; $P_j = qA'_j + dV'_j$ ; $j = 0, 1, 2, 3$.

Therefore

$$\begin{aligned}0 = & \ (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \{(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \Psi\} \\ \approx & \ [(p_0 + \hbar\partial_{g0})^2 - (p_1 + \hbar\partial_{g1})^2 - (p_2 + \hbar\partial_{g2})^2 - (p_3 + \hbar\partial_{g3})^2 - (P_0 + \hbar\partial_{e0})^2 \\ & - (P_1 + \hbar\partial_{e1})^2 - (P_2 + \hbar\partial_{e2})^2 - (P_3 + \hbar\partial_{e3})^2 + q\hbar\lozenge^* \circ \mathbb{A}'^* + d\hbar\lozenge^* \circ (\boldsymbol{I}_0 \circ \mathbb{V}')^*] \circ \Psi\end{aligned} \tag{33}$$

where, $\Psi = \mathbb{B}/\hbar$ is the wave function; $(q\hbar/d)(\lozenge^* \circ \mathbb{A}'^*)$ is the interplay term of the electromagnetic-electromagnetic subfield with the spin $(q\hbar/d)$.

The above equation can be used to describe the quantum characteristics of intermediate particles which possess the spin $(q\hbar/d)$, E charge and G charge. Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can deduce the wave equation and its conclusions about the interplays of ordinary matter and dark matter.

### C. Dirac-like equation

In the octonion space, the electromagnetic-gravitational subfield and electromagnetic-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E current and G current of the intermediate particle $N(d,q)$ are $(S_0^e, S_1^e, S_2^e, S_3^e)$ and $(S_0^g, S_1^g, S_2^g, S_3^g)$ respectively. The $N(d,q)$ is the mixture of the intermediate particles $\gamma_e^e$ and $\gamma_e^g$ . When $\mathbb{T}=0$, the wave equation of particle $N(d,q)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) = 0 \tag{34}$$

When the energy $\mathbb{W}=0$, the Dirac-like equation can be attained from the above equation

$$\hbar\lozenge^* \circ (\mathbb{B}/\hbar) = 0 \tag{35}$$

From the above equation, we can conclude that intermediate particles possess spin $(q\hbar/d)$ with no G charge nor E charge, and obtain the corresponding quantum equation. Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can deduce Dirac-like equation and its conclusions about the interplays of dark matter and ordinary matter.

## VI. QUANTIZATION OF DARK MATTER B

In the electromagnetic-gravitational field, we can research the interplays among the gravitational-gravitational subfield (ordinary matter field) with gravitational-electromagnetic subfield (dark matter field), and describe their quantum properties of the field source particle and intermediate particle.

### A. Dirac and Schrodinger equations

In the octonion space, the gravitational-gravitational subfield and gravitational-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E momentum and G momentum of the field source particle $N(m,e)$ are $(s_0^e, s_1^e, s_2^e, s_3^e)$ and $(s_0^g, s_1^g, s_2^g, s_3^g)$ respectively. The $N(m,e)$ is the mixture of the general charges $Q_g^g$ (the mass, $m$) and $Q_g^e$ (the dark matter charge, $e$). When $\mathbb{U}=0$, the wave equation of the particle $N(m,e)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (-\boldsymbol{I} \circ \mathbb{M}/\hbar) = 0 \tag{36}$$

Because of $|s_0^g| \gg |s_i^g|$ and $|s_0^e| \gg |s_i^e|$ , then $(i=1,2,3)$

$$\begin{aligned}
\mathbb{W} &= (\mathbb{B} + \alpha\lozenge)^* \circ \{(\mathbb{R} + k_{rx}\mathbb{X}) \circ \mathbb{P}\} \\
&\approx \alpha\lozenge^* \circ \left\{(\mathbb{R} + k_{rx}\mathbb{X}) \circ (\mu_g^g s_0^g + \mu_g^e \boldsymbol{I}_0 s_0^e)\right\}/\mu_g^g + \mathbb{B}^* \circ \mathbb{M} \\
&\approx \alpha(m\mathbb{V}' + e\mathbb{A}') + \mathbb{B}^* \circ \mathbb{M} + \alpha[\mu_g^e s_0^e \{\lozenge^* \circ (\mathbb{R} \circ \boldsymbol{I}_0)\} + \mu_g^g k_{rx} s_0^g \mathbb{A}]/\mu_g^g
\end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}\mu_g^e/\mu_g^g)\lozenge^* \circ (\mathbb{X} \circ \boldsymbol{I}_0) = a'_0 + \boldsymbol{i}_1 a'_1 + \boldsymbol{i}_2 a'_2 + \boldsymbol{i}_3 a'_3 + \boldsymbol{I}_0 A'_0 + \boldsymbol{I}_1 A'_1 + \boldsymbol{I}_2 A'_2 + \boldsymbol{I}_3 A'_3$; $s_0^g = mc, s_0^e = ec$; $\mathbb{V} = v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3 + \boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3$ ; $e$ is the $Q_g^e$, $m$ is the $Q_g^g$ .

When the $\mathbb{B}$ is small, and the sum of last two terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned}
\mathbb{W}/\alpha = &\ m\mathbb{V} + e\mathbb{A}' \\
= &\ (ea'_0 + mv_0) + \boldsymbol{i}_1(ea'_1 + mv_1) + \boldsymbol{i}_2(ea'_2 + mv_2) + \boldsymbol{i}_3(ea'_3 + mv_3) \\
&+ \boldsymbol{I}_0(eA'_0 + mV_0) + \boldsymbol{I}_1(eA'_1 + mV_1) + \boldsymbol{I}_2(eA'_2 + mV_2) + \boldsymbol{I}_3(eA'_3 + mV_3) \\
= &\ p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3
\end{aligned}$$

where, $p_j = ea'_j + mv_j$ ; $P_j = eA'_j + mV_j$ ; $j = 0,1,2,3$.

Therefore

$$\begin{aligned}
0 = &\ (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \{(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \Psi\} \\
\approx &\ [(p_0 + \hbar\partial_{g0})^2 - (p_1 + \hbar\partial_{g1})^2 - (p_2 + \hbar\partial_{g2})^2 - (p_3 + \hbar\partial_{g3})^2 - (P_0 + \hbar\partial_{e0})^2 \\
&- (P_1 + \hbar\partial_{e1})^2 - (P_2 + \hbar\partial_{e2})^2 - (P_3 + \hbar\partial_{e3})^2 + e\hbar\lozenge^* \circ \mathbb{A}'^* + m\hbar\lozenge^* \circ (\boldsymbol{I}_0 \circ \mathbb{V})^*] \circ \Psi
\end{aligned} \tag{37}$$

where, $\Psi = -\boldsymbol{I} \circ \mathbb{M}/\hbar$ is the wave function.

In the above equation, the conservation of wave function is influenced by the field potential, field strength, field source, velocity, charge and mass etc. When the linear momentum $P_j$ is equal approximately to 0, and the wave function is $\Psi = \psi(r)exp(-\boldsymbol{I}\,Et/\hbar)$, the above equation can be simplified as

$$\begin{aligned} 0 = & \quad [(p_0 - \boldsymbol{I}\,E/c)^2 - (p_1)^2 - (p_2)^2 - (p_3)^2 + e\hbar\lozenge^* \circ \mathbb{A}'^*] \circ \psi(r) \\ \approx & \quad [(ea'_0 - \boldsymbol{I}\,E/c) - (1/2mc)\left\{(p_1)^2 + (p_2)^2 + (p_3)^2\right\} + (e\hbar/2mc)(\lozenge^* \circ \mathbb{A}'^*)] \circ \psi(r) \end{aligned} \tag{38}$$

where, $E$ is the energy; $(e\hbar/2m)(\lozenge^* \circ \mathbb{A}'^*)$ is the interplay term of the gravitational-electromagnetic subfield with the spin $(e\hbar/2m)$.

Limited within certain conditions, Eq.(11) of the electromagnetic-gravitational field in the octonion space can deduce the Dirac and Schrodinger equations and their conclusions about interplays of dark matter and ordinary matter, including the 'spin' and 'magnetic moment' etc.

### B. Intermediate particle equations

In the octonion space, the gravitational-gravitational subfield and gravitational-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E momentum and G momentum of the field source particle $N(m,e)$ are $(s^e_0, s^e_1, s^e_2, s^e_3)$ and $(s^g_0, s^g_1, s^g_2, s^g_3)$ respectively. The $N(m,e)$ is the mixture of the intermediate particles $\gamma^g_g$ and $\gamma^e_g$ . When $\mathbb{T} = 0$, the wave equation of the particle $N(m,e)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) = 0 \tag{39}$$

When the $\mathbb{B}$ is small, and the sum of last two terms is equal approximately to zero, the above equation can be written as follows

$$\begin{aligned} \mathbb{W}/\alpha = & \quad m\mathbb{V} + e\mathbb{A}' \\ = & \quad (ea'_0 + mv_0) + \boldsymbol{i}_1(ea'_1 + mv_1) + \boldsymbol{i}_2(ea'_2 + mv_2) + \boldsymbol{i}_3(ea'_3 + mv_3) \\ & \quad + \boldsymbol{I}_0(eA'_0 + mV_0) + \boldsymbol{I}_1(eA'_1 + mV_1) + \boldsymbol{I}_2(eA'_2 + mV_2) + \boldsymbol{I}_3(eA'_3 + mV_3) \\ = & \quad p_0 + \boldsymbol{i}_1 p_1 + \boldsymbol{i}_2 p_2 + \boldsymbol{i}_3 p_3 + \boldsymbol{I}_0 P_0 + \boldsymbol{I}_1 P_1 + \boldsymbol{I}_2 P_2 + \boldsymbol{I}_3 P_3 \end{aligned}$$

where, $\mathbb{A}' = (ck_{rx}\mu^e_g/\mu^g_g)\lozenge^* \circ (\mathbb{X} \circ \boldsymbol{I}_0) = a'_0 + \boldsymbol{i}_1 a'_1 + \boldsymbol{i}_2 a'_2 + \boldsymbol{i}_3 a'_3 + \boldsymbol{I}_0 A'_0 + \boldsymbol{I}_1 A'_1 + \boldsymbol{I}_2 A'_2 + \boldsymbol{I}_3 A'_3$; $s^g_0 = mc, s^e_0 = ec$; $\mathbb{V} = v_0 + \boldsymbol{i}_1 v_1 + \boldsymbol{i}_2 v_2 + \boldsymbol{i}_3 v_3 + \boldsymbol{I}_0 V_0 + \boldsymbol{I}_1 V_1 + \boldsymbol{I}_2 V_2 + \boldsymbol{I}_3 V_3$ ; $p_j = ea'_j + mv_j$ ; $P_j = eA'_j + mV_j$ ; $j = 0,1,2,3$; $e$ is the $Q^e_g$, $m$ is the $Q^g_g$ .

Therefore

$$\begin{aligned} 0 = & \quad (\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \{(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ \Psi\} \\ \approx & \quad [(p_0 + \hbar\partial_{g0})^2 - (p_1 + \hbar\partial_{g1})^2 - (p_2 + \hbar\partial_{g2})^2 - (p_3 + \hbar\partial_{g3})^2 - (P_0 + \hbar\partial_{e0})^2 \\ & \quad -(P_1 + \hbar\partial_{e1})^2 - (P_2 + \hbar\partial_{e2})^2 - (P_3 + \hbar\partial_{e3})^2 + e\hbar\lozenge^* \circ \mathbb{A}'^* + m\hbar\lozenge^* \circ (\boldsymbol{I}_0 \circ \mathbb{V})^*] \circ \Psi \end{aligned} \tag{40}$$

where, $\Psi = \mathbb{B}/\hbar$ is the wave function; $(e\hbar/m)(\lozenge^* \circ \mathbb{A}'^*)$ is the interplay term of the gravitational-electromagnetic subfield with the 'spin' $(e\hbar/m)$.

The above equation can be used to describe the quantum characteristics of intermediate particles which possess the spin $(e\hbar/m)$, E mass and G mass. Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can deduce the wave equation and its conclusions about the interplays of ordinary matter and dark matter.

### C. Dirac-like equation

In the octonion space, the gravitational-gravitational subfield and gravitational-electromagnetic subfield are generated by the physical object $M$ which owns rotation and charge. The E momentum and G momentum of the field source particle $N(m,e)$ are $(s^e_0, s^e_1, s^e_2, s^e_3)$ and $(s^g_0, s^g_1, s^g_2, s^g_3)$ respectively. The $N(m,e)$ is the mixture of the intermediate particles $\gamma^g_g$ and $\gamma^e_g$ . When $\mathbb{T} = 0$, the wave equation of the particle $N(m,e)$ which moves around $M$ is

$$(\mathbb{W}/\alpha + \hbar\lozenge)^* \circ (\mathbb{B}/\hbar) = 0 \tag{41}$$

When the energy $\mathbb{W} = 0$, the Dirac-like equation can be attained from the above equation

$$\hbar\lozenge^* \circ (\mathbb{B}/\hbar) = 0 \tag{42}$$

From the above equation, we can conclude that intermediate particles possess the spin ($e\hbar/m$) with no G mass nor E mass, and obtain the corresponding quantum equation. Limited within certain conditions, Eq.(13) of the electromagnetic-gravitational field in the octonion space can deduce Dirac-like equation and its conclusions about the interplays of the dark matter and the ordinary matter.

## VII. CONCLUSIONS

The paper describes the quantization theory of electromagnetic and gravitational interactions and dark matter, including the Dirac equation, Schrodinger equation, and Dirac-like equation etc. And there exist the quantum interplays of ordinary matter and dark matter in the octonion space.

In the electromagnetic-gravitational subfield (electromagnetic field) and gravitational-gravitational subfield (gravitational field), we extend Dirac and Schrodinger equations of the field source particles, and deduce the Dirac-like equation of intermediate particles. It predicts that there exists one sort of field source particle (electric charge and mass), which possesses the spin ($q\hbar/2m$). And it also predicts that there exists one kind of intermediate particle, which is the mixture of the $\gamma_e^g$ (photon) and $\gamma_g^g$, and may possess the spin ($q\hbar/m$) with no G mass nor G charge.

In the electromagnetic-gravitational subfield (electromagnetic field) and electromagnetic-electromagnetic subfield (dark matter field), we infer the Dirac equation and Schrodinger equation of the field source particles, and deduce the Dirac-like equation of intermediate particles. It predicts that the dark matter field has one sort of field source particle (electric charge and E charge), which possesses the spin ($q\hbar/2d$). And it also predicts that the dark matter field has one sort of the intermediate particle, which is the mixture of the $\gamma_e^g$ (photon) and $\gamma_e^e$, and may possess the spin ($q\hbar/d$) with no E charge nor G charge.

In the gravitational-gravitational subfield (gravitational field) and gravitational-electromagnetic subfield (dark matter field), we extend Dirac and Schrodinger equations of the field source particles, and deduce the Dirac-like equation of intermediate particles. It predicts that there exists one sort of field source particle (G mass and E mass), which possesses the spin ($e\hbar/2m$). And it also predicts that there exists one kind of intermediate particle, which is the mixture of the $\gamma_g^e$ and $\gamma_g^g$, and may possess the spin ($e\hbar/m$) with no G mass nor E mass.

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[1] A. Bosma, *Celestial Mechanics & Dynamical Astronomy*, **72**, 69 (1998).
[2] V. C. Rubin, A. H. Waterman and J. D. P. Kenney, *Astronomical Journal*, **118**, 236 (1999).
[3] Z. Weng, arXiv:physics/0612102.
[4] S. L. Adler, *Quaternionic Quantum Mechanics and Quantum Fields*, (Oxford University Press, New York, 1995).
[5] N. Wu, *Commun. Theor. Phys.*, **45**, 452 (2006).
[6] N. Boulanger, F. Buisseret, P. Spindel, *Phys. Rev. D*, **74**, 125014 (2006).
[7] D. Singh, N. Mobed, G. Papini, *Phys. Rev. Lett.*, **97**, 041101 (2006).
[8] J. S. Lomont, *Phys. Rev.*, **111**, 1710 (1958).
[9] H. E. Moses, *Phys. Rev.*, **113**, 1670 (1959).
[10] G. Corvino, G. Montani, *Mod. Phys. Lett. A*, **19**, 2777 (2004).
[11] D. H. Oaknin, A. Zhitnitsky, *Phys. Rev. D*, **71**, 023519 (2005).
[12] C. R. Galley, B. L. Hu, S.-Y. Lin, *Phys. Rev. D*, **74**, 024017 (2006).